Sixth Form Algebra Induction Booklet

Contents

Introduction

This booklet has been designed to aid the transition of students entering the Sixth Form. Many areas of AS and A2 Mathematics require the confident use of algebraic concepts. Our experience has taught us that without such confidence, many students will struggle and perhaps not make the progress that they should.

It is hoped that the completion of this booklet will provide several benefits,

- The student's new teachers will be able to obtain an overview of the student's areas of strength and weakness;
- Mathematical skill levels will be maintained during the summer holiday;
- Students will consolidate and perhaps enhance their knowledge of key algebraic concepts.

At the back of this booklet you will find the exercises that need to be completed before September,

- **Single Mathematicians** should attempt sections A to E (inclusive); whilst
- **Double Mathematicians** should attempt *all* sections.
- All work should be completed on **A4 lined paper**, with your name clearly written at the top.
- Solutions need to be well-structured and presented in a neat, orderly manner.
- The solutions to most of the exercises appear at the back of the booklet; consequently, it is expected that students will produce *full* solutions and show all workings. Failure to do so may result in students being asked to **repeat the work**.
- It is also expected that students tick solutions that they complete successfully; thus allowing their teachers to focus on the areas of difficulty when the work is collected in.
- You are advised to work on this gradually through the Summer or to work through it in the final fortnight, so that your basic skills are refreshed in time for starting Year 12. If you complete the work too early, this will defeat the object of refreshing your skills after the long break.
- The completed work will be collected in **your** **<u>first Mathematics lesson in September.</u>**
- There will be an assessment of your algebraic skills in the first couple of weeks of term.

Please note that the successful completion of this booklet is a requirement for entry into Mathematics at St Olave's.

There are notes and examples contained within this booklet to help you. Some of the exercises are demanding. Do not panic. Persevere. You will feel the benefits later in the course.

Good luck,

Miss Meera Lawrence

Head of Mathematics

A. Expanding brackets

We often need to expand (multiply out) brackets in order to simplify an expression. Various methods may be employed, many people use the FOIL method (shown below), but the key point to remember is that everything on the *inside* needs to be multiplied by everything on the *outside*.

- $x(2x+3y^2) = 2x^2 + 3xy^2$

- Expand $(x+2)(x-3)$ using the FOIL method (First, Outside, Inside, Last)

$$(x+2)(x-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

With care, we can expand brackets containing any number of terms,

- $$\begin{aligned}(3a-b-c)(a+2b+3c) &= 3a(a+2b+3c) - b(a+2b+3c) - c(a+2b+3c) \\ &= 3a^2 + 6ab + 9ac - ab - 2b^2 - 3bc - ac - 2bc - 3c^2 \\ &= 3a^2 - 2b^2 - 3c^2 + 5ab - 5bc + 8ac\end{aligned}$$

Certain results are important and it is worth the effort to learn them…

- $(a+b)^2 = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

- $(a-b)^2 = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$

- $(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$

Therefore,

- $(3x-2y)^2 = (3x)^2 - 2(3x)(2y) + (2y)^2 = 9x^2 - 12xy + 4y^2$

- $(3x-2y)(3x+2y) = (3x)^2 - (2y)^2 = 9x^2 - 4y^2$

B. Factorising expressions

Whilst it is important to be able to expand brackets, it is possibly more important to be able to reverse the process; that is, to be able to factorise an expression.

Common factors

Some expressions can be factorised by identifying common factors.

- $3xy - 12x^2 = 3(xy - 4x^2) = 3x(y - 4x)$ *this expression has two common factors*

Four-term expressions

Some expressions can be factorised by grouping in pairs.

- $2ax + 3ay - 4bx - 6by = a(2x + 3y) - 2b(2x + 3y) = (a - 2b)(2x + 3y)$

$(2x + 3y)$ is now a common factor

Quadratics

Depending on the particular quadratic, the process of factorisation may be easy or difficult.

Using the difference of two squares

Be on the look out for these situations,

- $x^2 - 4y^2 = (x)^2 - (2y)^2 = (x + 2y)(x - 2y)$
- $8x^2 - 50 = 2(4x^2 - 25) = 2(2x + 5)(2x - 5)$

When the coefficient of x^2 is one

Simply find two numbers that *multiply* to give the constant and *sum* to give the coefficient of x

- $x^2 + x - 6 = (x + 3)(x - 2)$ *multiply to give –6 and add to give +1; i.e. 3 and –2*

When the coefficient of x^2 is *not* one

This is more difficult. For example, if we needed to factorise $\mathbf{4x^2 - 4x - 15}$, the solution could be of the form $(4x + ?)(x + ??)$ or $(2x + ?)(2x + ??)$. If you are lucky you might be able to spot the correct factorisation, but most people would have to resort to the following algorithm.

1. Multiply the coefficient of x^2 by the constant term $4 \times -15 = \mathbf{-60}$
2. Find factors of – 60 that sum to give the coefficient of x (i.e. – 4) $+6 - 10 = \mathbf{-4}$
3. Split the middle term using these numbers $4x^2 + \mathbf{6}x - \mathbf{10}x - 15$
4. Factorise the first two terms and then the last two terms $2x(2x + 3) - 5(2x + 3)$
5. Complete the factorisation… easy! $(2x - 5)(2x + 3)$

C. Quadratic equations

The ability to calculate the roots of a quadratic equation is extremely useful. Quadratic equations occur in the most unlikely areas of mathematics – the flight of a projectile, for example.

Please note that the problem may require you to rearrange an equation into the form $ax^2 + bx + c = 0$ *before* attempting to solve it.

$$x - 67 + 3x^2 = 9 + 6x - 2x^2$$
$$5x^2 - 5x - 76 = 0$$

Factorisation

We can use factorisation (see above for details) to solve $ax^2 + bx + c = 0$.

- Solve $x^2 + x - 6 = 0$
    - As we have seen above, $x^2 + x - 6 = 0$
      $$(x+3)(x-2) = 0$$
    - Now if the left-hand side is equal to zero, either $(x + 3) = 0$ or $(x - 2) = 0$
    - Therefore, the **roots** of the equations are $x = -3$ and $x = 2$.
    - The **solution** to the equation is the set $\{-3,2\}$; i.e. *all* roots to the equation.
- Solve $4x^2 - 4x - 15 = 0$
    - As we have seen above, $4x^2 - 4x - 15 = 0$
      $$(2x-5)(2x+3) = 0$$
    - Employing the same logic as before we see that the roots are $x = 2\frac{1}{2}$ and $x = -1\frac{1}{2}$

The quadratic formula

This can be used to solve quadratic equations by inputting the coefficients of $ax^2 + bx + c = 0$ into the following equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Solve $x^2 + 3x - 2 = 0$

Careful…

Here $a = 1$, $b = 3$ and $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - (4 \times 1 \times -2)}}{2 \times 1}$$
$$= \frac{-3 \pm \sqrt{17}}{2}$$
$$= 0.562 \text{ or } -3.56 \qquad (3sf)$$

D. Manipulating formulae

In the formula A = πr^2, A is the *subject* of the formula.

Single occurrences

- Make *a* the subject of *s* = *ut* + ½ *at*²

$$s = ut + \frac{1}{2}at^2$$

[a only occurs in one term ∴ isolate it]

$$a = 2\left(\frac{s - ut}{t^2}\right)$$

- Make *h* the subject of $S = \pi r\sqrt{h^2 + r^2}$

$$S = \pi r\sqrt{h^2 + r^2}$$

$$\frac{S}{\pi r} = \sqrt{h^2 + r^2}$$ [isolate the $\sqrt{\ }$]

$$\left(\frac{S}{\pi r}\right)^2 = h^2 + r^2$$ [square both sides]

$$h^2 = \left(\frac{S}{\pi r}\right)^2 - r^2$$

$$h = \sqrt{\left(\frac{S}{\pi r}\right)^2 - r^2}$$

Multiple occurrences

With multiple occurrences, collect all occurrences of the relevant variable on one side of the equation and factorise.

- Make *x* the subject of $y = \frac{x+1}{x+2}$

$$y = \frac{x+1}{x+2}$$

$$y(x+2) = x+1$$

$$xy + 2y = x+1$$

$$xy - x + 2y = 1$$

$$xy - x = 1 - 2y$$

All the *x*s are on one side; now we can factorise…

$$x(y-1) = 1 - 2y$$

$$x = \frac{1-2y}{y-1}$$

E. Indices

Definitions

In a^m, a is the **base** and m is the **index**. Please note that the plural of index is indices, not indic*es*.

Index laws

If two quantities are in the same base then the following rules apply:

$$a^m \times a^n = a^{(m+n)}$$
$$a^m \div a^n = \frac{a^m}{a^n} = a^{(m-n)}$$
$$(a^m)^n = a^{mn}$$
$$a^0 = 1$$
$$a^{-m} = \frac{1}{a^m}$$
$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Do not confuse these two rules…

Questions may require you to convert all quantities to the same base and/or combine several of the rules above.

- Find the value of

(a) $81^{\frac{1}{2}}$,

$$81^{\frac{1}{2}} = \sqrt{81} = 9$$

(b) $81^{\frac{3}{4}}$,

$$81^{\frac{3}{4}} = \left(\sqrt[4]{81}\right)^3 = (3)^3 = 27$$

(c) $81^{-\frac{3}{4}}$.

$$81^{-\frac{3}{4}} = \frac{1}{81^{\frac{3}{4}}} = \frac{1}{\left(\sqrt[4]{81}\right)^3} = \frac{1}{(3)^3} = \frac{1}{27}$$

- Evaluate $16^{-\frac{3}{4}}$

$$16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}}$$
$$= \frac{1}{\sqrt[4]{16^3}} = \frac{1}{(\sqrt[4]{16})^3}$$
$$= \frac{1}{2^3} = \frac{1}{8}$$

We can either cube 16 and *then* find the fourth-root; or we can find the fourth-root of 16 and cube the answer.

Obviously, one option is much easier than the other.

F. Surds

A surd is an *irrational* number. Often it includes the positive root of a non-square number; for example, √3 and (2 – √5) are surds, but √4 is not a surd, as √4 = 2.

Surd laws

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

When simplifying surds it is important to try and identify *square* factors.

Examples:

- Simplify the following

(a) $\sqrt{72}$

Square number

$$\sqrt{72} = \sqrt{36} \times \sqrt{2}$$
$$= 6\sqrt{2}$$

(b) $\sqrt{27} + 4\sqrt{12}$

$$\sqrt{27} + 4\sqrt{12} = \sqrt{9}\sqrt{3} + 4(\sqrt{4}\sqrt{3})$$
$$= 3\sqrt{3} + 4(2\sqrt{3})$$
$$= 3\sqrt{3} + 8\sqrt{3}$$
$$= 11\sqrt{3}$$

(c) $(4 + \sqrt{3})^2 - 2(1 - \sqrt{3})^2$

$$(4+\sqrt{2})^2 - 2(1-\sqrt{2})^2 = (16 + 4\sqrt{2} + 4\sqrt{2} + 2) - 2(1 - \sqrt{2} - \sqrt{2} + 2)$$
$$= (18 + 8\sqrt{2}) - 2(3 - 2\sqrt{2})$$
$$= 18 + 8\sqrt{2} - 6 + 4\sqrt{2}$$
$$= 12 + 12\sqrt{2}$$

Many people are confused by this simplification, but $8x + 4x = 12x$ whether x is rational or not…

Rationalising the denominator

It is preferable to have a rational denominator; therefore, if the denominator is irrational we must rationalise it.

Rationalise the denominators in the following quotients

By multiplying top *and* bottom by √3, we rationalise the denominator.

- $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
- $\frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$

G. Completing the square

The process of completing the square involves re-writing ($ax^2 + bx + c$) as $a(x + p)^2 + q$; that is, a 'square' plus an 'adjustment'.

The case when a = 1

For example, let's put the quadratic equation $x^2 - 4x - 3 = 0$ into completed square form.

- Clearly, if we wish to end up with $x^2 - 4x$, we need to begin with $(x - 2)^2$
- $(x - 2)^2 = x^2 - 4x \mathbf{+ 4}$, which is nearly the quadratic required
- However, we don't want '+ 4', we want '– 3' and so we must subtract 7 – this is the 'adjustment'.

$$x^2 - 4x - 3 = (x - 2)^2 - 7$$

If the coefficient of x^2 is *one* then the number in the bracket is half of the coefficient of x

- Complete the square for $x^2 + 6x + 1$

$$x^2 + 6x + 1 = 0$$
$$(x+3)^2 - 9 + 1 = 0$$
$$(x+3)^2 - 8 = 0$$

The case where a ≠ 1

In the case where a ≠ 1, we start by taking *a* out as a factor; then we complete the square for the quadratic inside the bracket; before finally multiplying out.

- Complete the square for $2x^2 - x + 1$

The first step is take out the coefficient of x^2 as a factor

$$2(x^2 - \tfrac{1}{2}x + \tfrac{1}{2})$$

Now we complete the square as before…

$$2\left(x^2 - \frac{1}{2}x + \frac{1}{2}\right) = 2\left[\left(x - \frac{1}{4}\right)^2 - \frac{1}{16} + \frac{1}{2}\right]$$
$$= 2\left[\left(x - \frac{1}{4}\right)^2 + \frac{7}{16}\right]$$
$$= 2\left(x - \frac{1}{4}\right)^2 + \frac{7}{8}$$

As before, this number is half the coefficient of x ; i.e. half of – ½

Rather than trying to work out the 'adjustment' in one go, simply subtract the constant from the square.

Finally, multiply out to leave the quadratic in completed square form.

H. Algebraic fractions

Algebraic fractions may be dealt with in the same way as numerical fractions. The key points to remember are:

- Factorise all numerators and denominators before proceeding; be on the look out for quadratics that can be factorised using the difference of two squares;
- For addition / subtraction, find the lowest common multiple (LCM) for the denominator;
- For multiplication, cancel common factors in the numerator and denominator *before* multiplying;
- For division, change to a multiplication sign, invert the second fraction and proceed as for multiplication.

Standard operations

- $\frac{2x^2+x}{4x^2-1} = \frac{x(2x+1)}{(2x-1)(2x+1)} = \frac{x}{2x-1}$ [cancelling the common factor]

- $\frac{x}{6x+3} + \frac{2}{x-2} = \frac{x}{3(2x+1)} + \frac{2}{x-2}$ [Factorise denominator]

$= \frac{x(x-2)}{3(2x+1)(x-2)} + \frac{2(3)(2x+1)}{3(2x+1)(x-2)}$ [LCM is $3(2x+1)(x-2)$]

$= \frac{x(x-2)+6(2x+1)}{3(2x+1)(x-2)}$ [combine fractions]

$= \frac{x^2+10x+6}{3(2x+1)(x-2)}$ [simplify]

$\frac{2x-1}{x^2-3x+2} - \frac{3x-8}{2x^2+x-3} = \frac{2x-1}{(x-1)(x-2)} - \frac{3x-8}{(2x+3)(x-1)}$ [Factorise]

$= \frac{(2x-1)(2x+3)}{(x-1)(x-2)(2x+3)} - \frac{(3x-8)(x-2)}{(2x+3)(x-2)(x-1)}$ [LCM is $(2x+3)(x-2)(x-1)$]

$= \frac{(4x^2+4x-3)-(3x^2-14x+16)}{(x-1)(x-2)(2x+3)}$ [Multiply out]

$= \frac{x^2+18x-19}{(x-1)(x-2)(2x+3)}$

$= \frac{(x-1)(x+19)}{(x-1)(x-2)(2x+3)}$ [Factorise]

$= \frac{x+19}{(x-2)(2x+3)}$ [Simplify]

- $$\frac{x^2-1}{x^2+5x+6}\times\frac{x^2-4}{x^2+4x+3}=\frac{(x-1)(x+1)}{(x+2)(x+3)}\times\frac{(x-2)(x+2)}{(x+1)(x+3)}$$
$$=\frac{(x-1)}{(x+3)}\times\frac{(x-2)}{(x+3)} \quad \text{[cancel common factors]}$$
$$=\frac{(x-1)(x-2)}{(x+3)^2}$$

- $$\frac{x^2+x-6}{x^2+x-2}\div\frac{x^2-9}{x^2+3x-4}=\frac{(x+3)(x-2)}{(x+2)(x-1)}\div\frac{(x+3)(x-3)}{(x+4)(x-1)} \quad \text{[Factorise]}$$
$$=\frac{(x+3)(x-2)}{(x+2)(x-1)}\times\frac{(x+4)(x-1)}{(x+3)(x-3)} \quad \text{[Invert]}$$
$$=\frac{(x-2)}{(x+2)}\times\frac{(x+4)}{(x-3)} \quad \text{[cancel]}$$
$$=\frac{(x-2)(x+4)}{(x+2)(x-3)} \quad \text{[simplify]}$$

Solving equations involving algebraic fractions

When solving equations involving algebraic fractions, multiply through by suitable factors to remove the denominators.

- $$\frac{x}{x+2}+\frac{x-1}{2x+1}=1 \quad \text{[Multiply both sides by } (x+2) \text{ and } (2x+1)\text{]}$$
$$x(2x+1)+(x-1)(x+2)=(x+2)(2x+1)$$
$$(2x^2+x)+(x^2+x-2)=2x^2+5x+2$$
$$x^2-3x-4=0 \quad \text{[Form the quadratic]}$$
$$(x-4)(x+1)=0$$
$$x=4 \text{ or } -1$$

I. Simultaneous equations

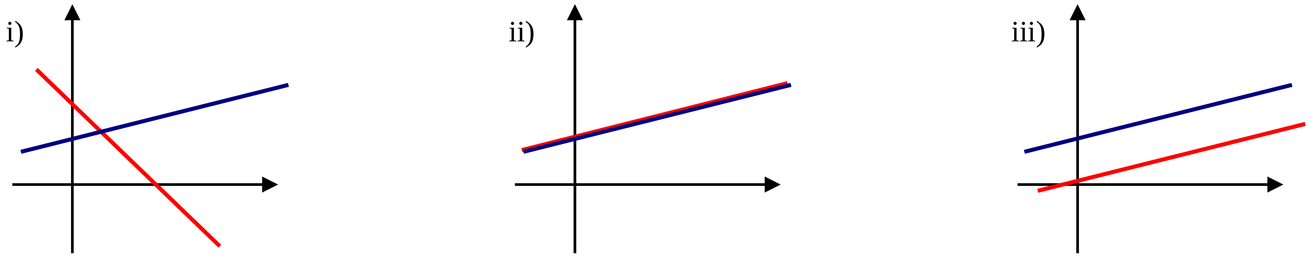
A linear equation in the form $y = mx + c$ represents a straight line with gradient m and y-intercept $(0, c)$.

If we have *two* such linear equations, which are simultaneously true, then there are three possible outcomes.

i). There may be a **unique solution**; the lines intersect at one point.

ii). There may be an **infinite number** of solutions; both equations refer to the same line.

iii). There is **no solution**; the lines are parallel.



A simultaneous equation may be solved by either:

Elimination

- Multiply one equation to ensure that you have the same number of *xs* or *ys* in each equation, then add or subtract as required.
- For example, solve $3x + 2y = 4$ and $2x + 5y = -1$

$$\begin{aligned} & 3x + 2y = 4 && \text{(A)} \\ & 2x + 5y = -1 && \text{(B)} \end{aligned}$$

$$\begin{aligned} 2\times(\text{A}) \quad & 6x + 4y = 8 && \text{(C)} \\ 3\times(\text{B}) \quad & 6x + 15y = -3 && \text{(D)} \end{aligned}$$

$$(\text{D}) - (\text{C}) \qquad 11y = -11$$

$$\begin{aligned} y &= -1 \\ x &= 2 \end{aligned}$$

Substitution

- Rearrange one of the equations to make either *x* or *y* the subject, then substitute this expression into the other equation.
- For example, solve $3x + 2y = 4$ and $2x + 5y = -1$

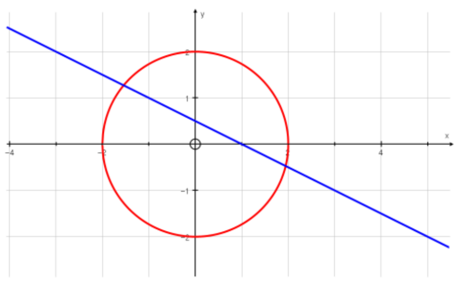
$$x = \frac{4-2y}{3}$$
$$2\left(\frac{4-2y}{3}\right) + 5y = -1$$
$$\frac{8}{3} - \frac{4}{3}y + 5y = -1$$
$$\frac{11}{3}y = -\frac{11}{3}$$
$$11y = -11$$
$$y = -1$$
$$x = 2$$

Non-linear simultaneous equations

- The method of substitution is the method to use to solve a pair of simultaneous equations when one of the equations is non-linear.
  - For example, solve the simultaneous equations $x^2 + y^2 = 4$ and $x + 2y = 1$
  - Here we could make either *x* or *y* the subject of the linear equation. Obviously, if we choose to make *x* the subject, it will result in a much easier equation.

$$x + 2y = 1$$
$$x = 1 - 2y$$
$$x^2 + y^2 = 4$$
$$(1-2y)^2 + y^2 = 4$$
$$1 - 4y + 4y^2 + y^2 = 4$$
$$5y^2 - 4y - 3 = 0$$
$$y = \frac{4 \pm \sqrt{16+60}}{10}$$
$$y = 1.27 \text{ or } -0.471 \qquad (3sf)$$

Graphical depiction of solution



Having found one of the variables it is VERY important that you substitute back into the LINEAR equation to find the corresponding values of the other variable.

When $y = 1.27$ $x = -1.54$
When $y = -0.471$ $x = 1.94 (3sf)$

You must state which *x* goes with which *y*.

J. Inequalities

Linear inequalities

A linear *inequality* can be treated as a linear *equation* with one important exception – if you multiply / divide an inequality by a negative quantity, the sign of the inequality reverses.

For example, it is true that 3 > 2 but it would not be true to say – 3 > – 2; this is why the sign must be reversed.

Example

- Solve $3x + 2 \geq 4 - x$
    - Remember, if you begin with an 'or equals to' inequality then you must end up with an 'or equals to' inequality, not a strict equality (≥ → ≥, > → >, etc, BUT ≥ $\not\to$ >)
    - Hopefully, you will not need reminding that a question involving an *inequality* NEVER EVER ends with *x* ***equals***…
    - $3x + 2 \geq 4 - x$

      $4x + 2 \geq 4$

      $4x \geq 2$

      $x \geq \frac{1}{2}$

Not $x = ½$

Quadratic inequalities

A quadratic inequality may also be treated as a quadratic equation with the normal exceptions. It is important to realise, however, that the resulting answer will one of two possible forms:

- $a < x < b$
- $x < a$ or $x > b$

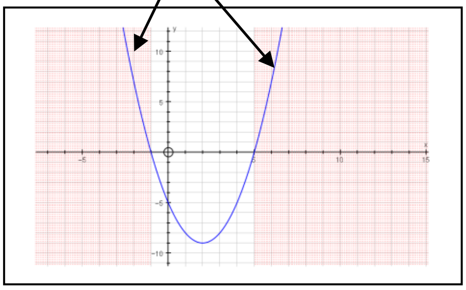
No other form is acceptable!!!

Often students invent their own notation – such as 3 > *x* > 5, which suggests that 3 > 5, they *mean* *x* < 3 and *x* > 5. Only the two forms given above are permissible.

Examples

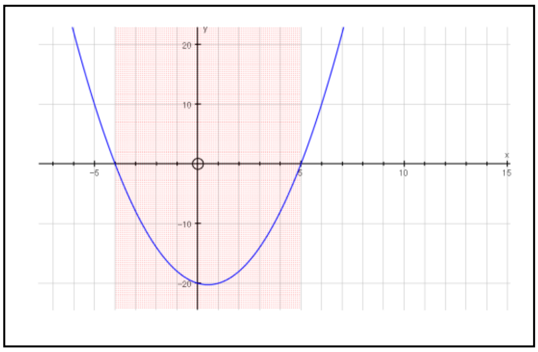
- Solve $x^2 - 4x - 5 > 0$
    - Factorising gives $(x - 5)(x + 1) > 0$
    - Clearly $x = -1$ and $x = 5$ are critical values
    - $x^2 - 4x - 5$ is a positive (☺) parabola.
    - We require the section that is greater than zero; i.e. the section *above* the *x*-axis
    - Therefore, we require the 'outside' area;
    - viz. $x < -1$ and $x > 5$.

The curve is > 0 in the shaded region

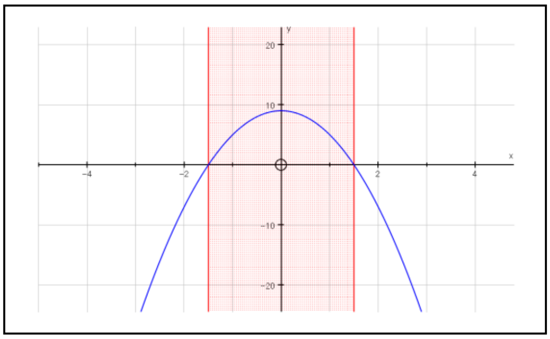


Look at the inequality sign…

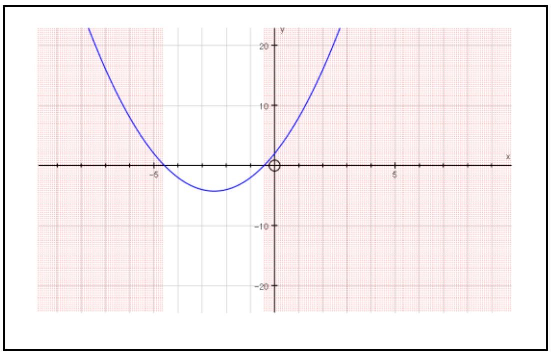
- Solve $x^2 - x - 20 \le 0$
  - Factorising gives $(x - 5)(x + 4) \le 0$
  - The critical values are $x = 5$ and $x = -4$
  - This is a positive (☺) parabola
  - We require the section *beneath* the *x*-axis
  - Therefore, we choose the 'middle' section
  - That is, $-4 \le x \le 5$



- Solve $9 - 4x^2 \ge 0$
  - Factorising gives $(3 - 2x)(3 + 2x) \ge 0$
  - The critical values are ± 1 ½
  - This is a negative (☹) parabola
  - We require the section *above* the *x*-axis
  - Therefore, we choose the 'middle' section
  - That is, $-1\frac{1}{2} \le x \le 1\frac{1}{2}$



- Solve $x^2 + 5 \ge 3 - 5x$
  - Rearrange to give $x^2 + 5x + 2 \ge 0$
  - This cannot be factorised
  - Therefore use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - The critical values are $x = \frac{-5 \pm \sqrt{17}}{2}$
  - This is a positive (☺) parabola
  - We require the section *above* the *x*-axis
  - Therefore, $x \le \frac{-5 - \sqrt{17}}{2}$ and $x \ge \frac{-5 + \sqrt{17}}{2}$



As you can see, it is not always clear which area you require. It is very important to draw a sketch of the graph to avoid making silly mistakes.

Assessment questions

A. Expanding brackets

Expand and simplify the following. **NB *no* answers given.**

1. $(x+2)(x-5)$
2. $(3x-2y)(2x+y)$
3. $(2x+1)(2x-1)$
4. $(a+3b)^2$
5. $(4x-7)(2x+3)$
6. $(4x-7)(3x-1)$
7. $(a-b+c)(2a+b-2c)$
8. $(5x-9)^2$
9. $(x^2+7)(x^2-7)$
10. $(2x^2+x+2)(x^2-3x-4)$

B. Factorising expressions

Factorise the following expression. **NB *no* answers given.**

1. x^2+6x+5
2. $x^2-8x-20$
3. $t^2+5t-36$
4. $ad+bd-3ac-3bc$
5. $2x^2+11x+15$
6. $5x^2-17x+6$
7. $3x^2-7x-6$
8. y^2-64
9. $15+x-2x^2$
10. $5x^2-125$
11. $9x^2-12xy+4y^2$
12. $(x+1)^2-y^2$
13. x^4-8x^2-9
14. $6x^2-19x+10$
15. $p^2-q^2-5p+5q$

C. Quadratic equations

Solve the following equations. Give answers to 3sf where appropriate.

NB answers now given on final page.

1. $x^2+5x+6=0$
2. $x^2-8=2x$
3. $2x^2+x-3=0$
4. $2x^2-11x+15=0$
5. $6x^2+13x+6=0$
6. $x^2+7x+2=0$
7. $2x^2+3x=8$
8. $\frac{2}{x}=3+2x$
9. $2x+1=\frac{13}{x+2}$
10. $x^4-17x^2+16=0$

D. Manipulating formulae

Make the letter in the [bracket] the subject of the formulae.

1. $v^2=u^2+2as$ $[a]$
2. $s=ut+\frac{1}{2}at^2$ $[u]$
3. $T=2\pi\sqrt{\frac{l}{g}}$ $[l]$
4. $S=\frac{n}{2}[2a+(n+1)d]$ $[d]$
5. $S=\frac{a}{1-r}$ $[r]$
6. $y=\frac{1-x}{2x-3}$ $[x]$
7. $e=\frac{P-p}{PT-pt}$ $[p]$
8. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ $[u]$

E. Indices

Evaluate the following

1. 5^{-3}
2. $16^{\frac{1}{4}}$
3. $8^{-\frac{2}{3}}$
4. $\frac{10^8\times10^{-3}}{10^{-1}}$
5. $(4^{-5})^{\frac{1}{2}}$
6. $3a^{-4}\times2a^{-2}$
7. $\frac{(9a)^2}{(3a)^4}$
8. $\frac{6x\times12x^5}{8x^{-3}}$
9. $(16x^{-4})^{\frac{3}{4}}$
10. $\left(x\sqrt{x}\right)^4\div\sqrt{x}$

Double Mathematicians only, from this point forward.

F. Surds

Simplify the following. Full workings are expected.

1. $\sqrt{28}$
2. $\sqrt{54}$
3. $\sqrt{32} - \sqrt{8}$
4. $\sqrt{11} + \sqrt{44} - \sqrt{99}$
5. $\sqrt{18} \times \sqrt{8}$
6. $\frac{21}{\sqrt{7}}$
7. $\frac{16}{\sqrt{32}}$
8. $\frac{3\sqrt{12}}{2\sqrt{24}}$
9. $(3\sqrt{2})^3$
10. $\sqrt{12\frac{1}{4}}$

G. Completing the square

Complete the square for the following

1. $x^2 - 8x + 9$
2. $x^2 + 10x$
3. $x^2 - 3x + 3$
4. $2x^2 - 12x - 3$
5. $5x^2 - 25x + 9$
6. $3 + 2x - x^2$

H. Algebraic Fractions

Simplify Q1 to Q5; Solve the equations in Q6 and Q7

1. $\frac{4}{x+3} + \frac{5}{x-5}$
2. $\frac{2}{3x+5} - \frac{3}{6x-1}$
3. $\frac{2x^2-9x+4}{x^2-2x-8}$
4. $\frac{2}{x+3} - \frac{x-6}{x^2-9}$
5. $\frac{2x^2-5x+3}{x^2-1} \times \frac{2x^2+7x+6}{4x^2-9}$
6. $\frac{8}{2x-3} + \frac{3}{x+1} = 4$
7. $\frac{1}{2x-1} - \frac{3}{6x-1} = 2$

I. Simultaneous equations

Solve the following equations.

1. $2x + 5y = 11$, $4x + 3y = 29$
2. $4x - 3y = 1$, $6x + 2y = -5$
3. $y = 3x - 7$, $y = 3 - 5x$
4. $2x + y = 1$, $4x^2 + y^2 = 61$
5. $x - 3y = 13$, $xy + 12 = 0$
6. $xy = 30$, $3x + y = 21$
7. $2x + 3y = 1$, $4x^2 - 9y^2 = -17$
8. $x - 3y = 1$, $x^2 - 2xy - y^2 = 7$

J. Inequalities

Solve the following inequalities

1. $3(1 - 2x) > 2(2x + 1)$
2. $\frac{x}{3} \geq \frac{x}{4} + 1$
3. $(2x + 1)(x - 4) < 0$
4. $x^2 + 3x - 4 \geq 0$
5. $2x^2 + 3x < 0$
6. $4x^2 > 9$

Solutions

A. Expanding brackets

None given.

B. Factorising expressions

None given.

C. Quadratic equations

1. −3, −2
2. −2, 4
3. – 3/2, 1
4. 5/2, 3
5. – 3/2, – 2/3
6. – 6.70, – 0.298
7. – 2.89, 1.39
8. – 2, 1/2
9. – 3.91, 1.41
10. – 4, 4, −1, 1

D. Manipulating formulae

1. $a = \frac{v^2 - u^2}{2s}$
2. $u = \frac{2s - at^2}{2t}$
3. $l = \frac{gT^2}{4\pi^2}$
4. $d = \frac{2(S - an)}{n(n+1)}$
5. $r = \frac{S - a}{S}$
6. $x = \frac{1+3y}{1+2y}$
7. $p = \frac{P(eT-1)}{et-1}$
8. $u = \frac{fv}{v-f}$

E. Indices

1. 1/125
2. 2
3. 1/4
4. 1 000 000
5. 1/32
6. $6a^{-6}$
7. a^{-2}
8. $9x^9$
9. $8x^{-3}$
10. $x^{11/2}$

F. Surds

1. 2 √7
2. 3 √6
3. 2 √2
4. 0
5. 12
6. 3 √7
7. 2 √2
8. (3 √2) / 4
9. 54 √2
10. 7 / 2

G. Completing the square

1. $(x-4)^2 - 7$
2. $(x+5)^2 - 25$
3. $(x-1.5)^2 + 0.75$
4. $2(x-3)^2 - 21$
5. $5(x-2.5)^2 - 89/4$
6. $-(x-1)^2 + 4$

H. Algebraic fractions

1. $\frac{9x-5}{(x+3)(x-5)}$
2. $\frac{3x-17}{(3x+5)(6x-1)}$
3. $\frac{2x-1}{x+2}$
4. $\frac{x}{(x+3)(x-3)}$
5. $\frac{x+2}{x+1}$
6. – 0.5, 2.75
7. 0, 2/3

I. Simultaneous equations

1. (8, −1)
2. (– 0.5 , −1)
3. (1.25, – 3.25)
4. (3, −5) (−2.5, 6)
5. (4, −3) (9, – 4/3)
6. (2, 15) (5, 6)
7. (– 4, 3)
8. (4, 1) (−8, −3)

J. Inequalities

1. $x < 0.1$
2. $x \geq 12$
3. $-0.5 < x < 4$
4. $x \leq -4$, $x \geq 1$
5. $-1.5 < x < 0$
6. $x < -1.5$, $x > 1.5$